

Mathematics Design Collaborative
State of Georgia Department of Education





Solving Right Triangles in Applied Problems

Right Triangles in Your Environment

INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

-  Use trigonometric ratios to find sides and angles of right triangles.
-  Diagram and solve applied problems.

GEORGIA STANDARDS OF EXCELLENCE

This lesson involves mathematical content in the standards from across the grades, with emphasis on:

SMP 1 Make sense of problems and persevere in solving them.

SMP 3 Construct viable arguments and critique the reasoning of others.

SMP 4 Model with mathematics.

SMP 6 Attend to precision.

MGSE9-12.G.SRT.8 – Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

INTRODUCTION

This lesson is structured in the following way:

Before the Lesson: Students work individually, on an assessment task that is designed to reveal their current levels of understanding and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.

At the Start of the Lesson: Teacher will lead a whole-class interactive introduction.

During the Lesson: Students work collaboratively, in pairs, to complete a matching activity. After discussion with another group, each pair will perform an error analysis of four completed problems.

Whole-Group Class Discussion: In a plenary discussion students review the main math concepts of the lesson.




After the Whole-Group Class Discussion: Students will receive their pre-assessments back and will use them to complete a post-assessment that is similar.

MATERIALS REQUIRED

Each individual student will need:

-  Copy of the pre-assessment, Right Triangles in Your Environment
-  Copy of the post-assessment, Right Triangles in Your Environment - Revisited

Each pair of students will need:

-  Set of Situation Cards
-  Set of Diagram Cards
-  Error Analysis Sheets (4)

There are some projector resources to support whole-class discussions.

TEACHER PREP REQUIRED

Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made:

Copies should be made according to the guidelines above. There is no need to cut the cards apart prior to class because the cards are not in order. Students can cut the cards apart.

TIME NEEDED:

For Pre- 15 minutes For Lesson: 50 minutes For Post: 15 minutes
Assessment:

Special Note(s) about timing: The 50 minute lesson may need to be split into two periods if your students need longer to perform error analysis.

FRAMING FOR THE TEACHER:

Students often have misconceptions when working with applied problems involving angle of depression and angle of elevation. This unit attempts to identify two common misconceptions: 1) improperly choosing trigonometric functions and 2) labeling the wrong angle for angle of depression or elevation. The ability to solve applied problems using trigonometric functions is necessary in many real world jobs such as Coast Guard rescue, forest fire detection, aircraft navigation, etc. Note that the problems in this FAL may include mixed units. Students need to pay attention to detail when solving.

FRAMING FOR THE STUDENTS:

Say to the students:

This activity will take about 2 days for us to complete.

The reason we are doing this is to be sure that you understand: how to diagram and solve applied problems using trigonometric functions.

You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!

PRE-ASSESSMENT BEFORE THE LESSON

ASSESSMENT TASK:

Name of Assessment Task: *Right Triangles in Your Environment*

Time This Should Take: 15 minutes

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Right Triangles in Your Environment*

Briefly introduce the task and help the class to understand the problem and its context.

Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don't worry if you can't complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

Right Triangles in Your Environment

Little Lucy climbed an apple tree in her backyard. She knows that the tree is 30 feet from her mother's rose garden. Lucy looks down and sees the rose garden. She estimates that the angle of elevation is 40° .

1. Draw a diagram to show this situation. Label what you know.
2. Find how high Lucy is in the tree to the nearest foot. Show your calculations.

3. Silly Sammy used the diagram above to find the following: $\sin A = \frac{12}{13}$. Do you agree or disagree? Explain your reasoning.
4. Happy Hank used the same diagram. He stated the following: $\tan B = \frac{5}{12}$. Do you agree or disagree? Explain your reasoning.

COLLABORATION TIME/READING STUDENTS RESPONSES

You Will Not “Grade” These!

Collect students’ responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!)

You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point.

GUIDING QUESTIONS

<i>Common Issues:</i>	<i>Suggested questions and prompts:</i>
<i>Student has difficulty constructing a model for the situation.</i>	<i>What is the difference between horizontal and vertical? What are you trying to find in this situation? Describe the situation to me as you draw it on the board.</i>
<i>Student has difficulty labeling angles of the triangle.</i>	<i>What is the difference between an angle of elevation and an angle of depression?</i> <i><u>For depression:</u> Look straight ahead, keep your head still, but lower your eyes to see my feet. Can you show me with your arm what your eyes had to do?</i> <i>Can you show me that angle on your diagram?</i> <i><u>For elevation:</u> Look straight ahead, keep your head still, and raise your eyes so you are looking at the ceiling. Can you show me with your arm what your eyes did? Can you show me that angle on your diagram?</i>
<i>Student uses incorrect trigonometric function to evaluate sides and/or angles.</i>	<i>Can you tell me your procedure for finding this side or this angle?</i> <i>Can you show me where the opposite side, adjacent side, and hypotenuse are for this angle?</i> <i>Can you explain to me the difference between \sin and \sin^{-1}?</i>
<i>Student work appears correct, but answers do not make sense or appear wrong.</i>	<i>Is the calculator in the correct mode?</i> <i>Can you show me your keystrokes on the calculator?</i>

LESSON DAY

Part 1: Whole-Class Introduction:

Time to Allot: (15 minutes)

Give each student either a mini-whiteboard, pen, and eraser; or a sheet of unlined paper.

Use Slide 1 of the Projector Resources to project Situation 1 as you read it aloud. Each student should construct a diagram that models the situation.

Show your diagram to a classmate. Discuss any differences and similarities.

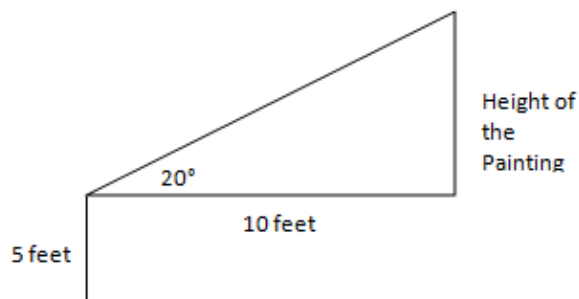
Students may need leading through the diagram to correctly label the angle of elevation. Use student responses and explanations to re-direct struggling students to understand the idea of an angle of elevation. If a student still does not understand, try the following:

Look straight ahead at the board (or wall). Keep your head completely still. Raise your eyes until you see the ceiling. Use your arm to show the movement of your eyes. The angle that your eyes move is the angle of elevation.

Instruct students to solve for the height of the painting on the white board. Students will need calculators for this part.

Show your answer to a classmate. Compare how you calculated the answer.

Some students may need to label the opposite, adjacent, and hypotenuse for the triangle. Students will also need a reminder about SOHCAHTOA. Lead students through the calculations.



$$\tan 20 = \frac{x}{10}$$

$$10 \tan 20 = x$$

$$3.6 \text{ feet}$$

Remember to add the 5 feet for the height of your eyes:

$$3.6 + 5 = 8.6 \text{ feet}$$

Situation 1

You are looking at a painting on the wall at the High Museum in Atlanta. You are standing 10 feet from the wall. Your angle of elevation to view the painting is 20°. (Your eyes are about 5 feet above the floor.)

Find the height of the painting.

Use Slide 2 of the Projector Resources to project Situation 2 as you read it aloud. Each student should construct a diagram that models the situation.

Show your diagram to a classmate. Discuss any differences and similarities.

Students may need leading through the diagram to correctly label the angle of depression. Use student responses and explanations to re-direct struggling students to understand the idea of an angle of elevation. If a student still does not understand, try the following:

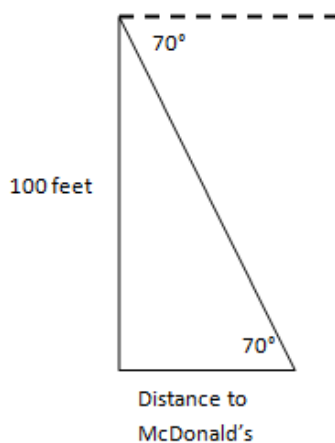
Look straight ahead at the board (or wall). Keep your head completely still. Lower your eyes until you see the floor. Use your arm to show the movement of your eyes. The angle that your eyes move is the angle of depression.

Use a diagram to demonstrate how the angle of depression is outside the triangle. Using either alternate interior angles or complementary angles, explain that the angle on the bottom is congruent to the angle of depression.

Instruct students to solve for the distance to the McDonald's on the white board. Students will need calculators for this part.

Show your answer to a classmate. Compare how you calculated the answer.

Some students may need to label the opposite, adjacent, and hypotenuse for the triangle. Lead students through the calculations.



$$\begin{aligned}\tan 70 &= \frac{100}{x} \\ x &= \frac{100}{\tan 70} \\ &36.4 \text{ feet}\end{aligned}$$

Situation 2

You are looking 120 feet above the street on the rooftop of your hotel. You look down and see a McDonald's restaurant. The angle of depression is 70°.

Find the distance to McDonald's.

G09.U2

Part 2: Collaborative Activity:**Time to Allot: (35 minutes)**

Put students into their pairs according to your analysis of student errors.

Do/Say the Following:

You will receive two sets of cards – one with situations and one with diagrams. Spend a few minutes to match each situation to its diagram.

After you have your cards matched, compare with another group. Discuss differences and try to reach an agreement.

When you have reached an agreement, I will listen to both groups explain their matches. You will then receive solutions for four of these situations. Work with your partner to do an error analysis of each solution. Make any necessary corrections. Explain why you made corrections.

During the Collaborative Activity, the Teacher has 3 tasks:

- 👤 Circulate to students' whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
- 👤 Circulate to other students also to support their reason in the same way.
- 👤 Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!

Part 3: Plenary (Summary) Discussion:**Time to Allot: (10 minutes)**

Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches.

NOTE: “Scribing” helps to increase student buy-in and participation. When a student answers a question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!

In the summary discussion, present each situation and discuss the mistake or mistakes.

You could extend this activity further by assigning the other 4 problems to be solved and allowing students to exchange and do error analysis on those problems.

Part 4: Improving Solutions to the Assessment Time to Allot: (15 minutes)**Right Triangles in Your Environment - Revisited**

The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to **UNGRADED FORMATIVE** assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.

Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds and kills misconceptions!

Return to the students their original assessment *Combining Inequalities*, as well as a second blank copy of the task. Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

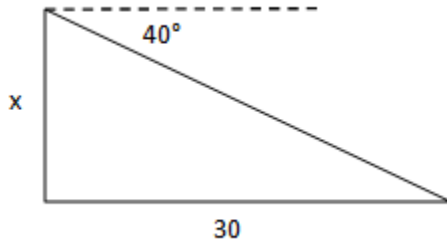
Explain to students that Questions 1 and 2 are concerned with just the first three clues. When answering these questions they should ignore Clues 4 and 5.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

PRE-ASSESSMENT (Answer Key)

Name of Assessment Task: Right Triangles in Your Environment

1. The diagram:

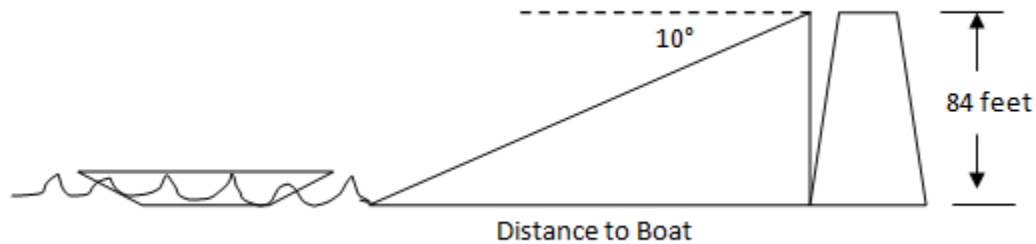


2. $x = 30 \tan 40$ x is 25 feet
3. Disagree. Sharp Sammy used the adjacent side instead of the opposite side. The answer should be $\frac{5}{13}$.
4. Disagree. Happy Hawk reversed the sides for tangent. He used adjacent over opposite instead of opposite over adjacent. The answer should be $\frac{12}{5}$.

POST-ASSESSMENT (Answer Key)

Name of Assessment Task: Right Triangles in Your Environment - Revisited

1. The diagram:



2. $x = 84/\tan 10$ x is 476 feet
3. Disagree. Angry Arthur reversed the hypotenuse and adjacent. The answer should be $\frac{12}{13}$.
4. Disagree. Beautiful Betty used adjacent instead of opposite for the top and then used the hypotenuse instead of adjacent for the bottom. The answer should be $\frac{12}{5}$.

Collaborative Activity - Matching (Answer Key)

A	2
B	1
C	8
D	5
E	7
F	6
G	3
H	4

B. Bend in Tree Error Analysis

Mark used the entire 100 feet for the diagonal section of the tree. He did convert the 2 yards to 6 feet. He should have then subtracted the 6 feet from the 100 feet yielding 94 feet for the diagonal.

Therefore, his work would be: $\cos^{-1}(6/94) = 86.3^\circ$

D. Duck, Duck, Goose Error Analysis

Kelly solved for the angle marked x correctly. However, she incorrectly identified the angle of depression. The answer should be: $x=38.7^\circ$

E. Emergency Error Analysis

Evan did not use the correct trig function. He used cos when he should have used sin. Therefore, his work should have been: $x = 100 \sin 20$ which gives an answer of: $x = 34.2$ feet

Evan also forgot about the fact that the ladder is mounted 2 yards above the ground. He should convert the 2 yards to 6 feet and add that to his previous answer. His final answer should be: 40.2 feet

H. Happy Landing Error Analysis

Mollie used the correct trig function, but switched the top and bottom. Tan should be opposite over adjacent.

Therefore, her work should be: $x = 20 / \tan 16$ which gives an answer of: $x = 69.7$ km

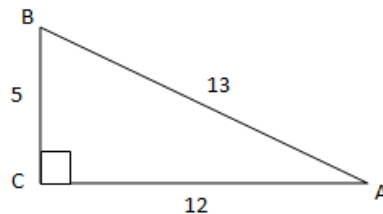
Secondly, Mollie incorrectly converted the km to miles. Since 1.6 km is equal to 1 mile, you must divide the answer by 1.6 instead of multiplying by 1.6 as Mollie did. See the ladder approach below:

$$\frac{69.7 \text{ km}}{1.6 \text{ km}} = 43.6 \text{ miles}$$

Right Triangles in Your Environment

Little Lucy climbed an apple tree in her backyard. She knows that the tree is 30 feet from her mother's rose garden. Lucy looks down and sees the rose garden. She estimates that the angle of elevation is 40° .

1. Draw a diagram to show this situation. Label what you know.
2. Find how high Lucy is in the tree to the nearest foot. Show your calculations.

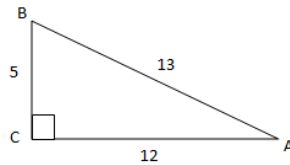


3. Silly Sammy used the diagram above to find the following: $\sin A = \frac{12}{13}$
Do you agree or disagree? Explain your reasoning.
4. Happy Hank used the same diagram. He stated the following: $\tan B = \frac{5}{12}$
Do you agree or disagree? Explain your reasoning.

Right Triangles in Your Environment – Revisited

Sailor Sam lives in a lighthouse on Cumberland Island. He looks out at the Atlantic Ocean and spots a boat in trouble. He needs to notify the Coast Guard of the location of the distressed boat. Using his clinometers (instrument to measure angles of elevation or depression), he notes that the angle of depression is 10° . He knows that he is 84 feet above the ocean.

1. Draw a diagram to show this situation. Label what you know.
2. Find the distance of the boat from the lighthouse to the nearest foot. Show your calculations.

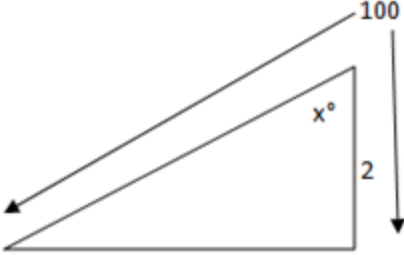
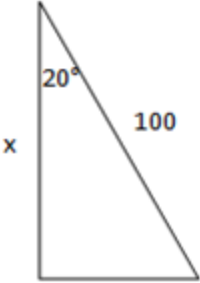
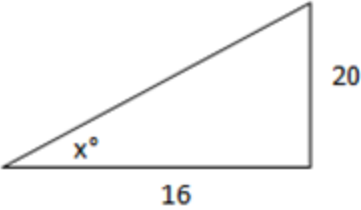
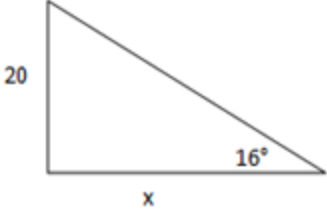
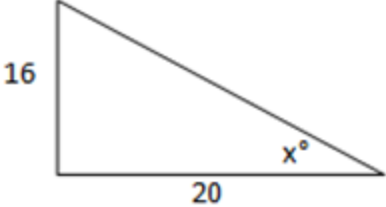
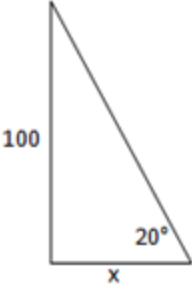
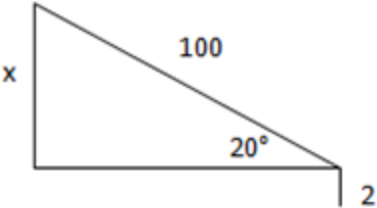
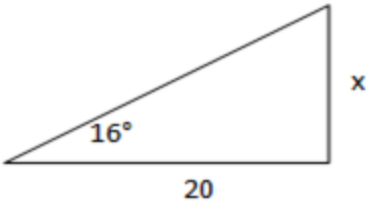


3. Angry Arthur used the diagram above to find the following: $\cos A = \frac{13}{12}$
Do you agree or disagree? Explain your reasoning.
4. Beautiful Betty used the same diagram. She stated the following: $\tan B = \frac{5}{13}$
Do you agree or disagree? Explain your reasoning.

Situation Cards for Matching Activity

<p>C Cesna Flight</p> <p>A small Cesna plane takes off from an airport and rises uniformly at an angle of 16° with the horizontal ground. After it has traveled over a horizontal distance of 20 miles, what is the altitude of the plane to the nearest foot?</p>	<p>G Good Morning</p> <p>From the tip of a shadow by the vertical object such as a tree, the angle of elevation of the top of the object is the same as the angle of elevation of the sun. What is the angle of elevation of the sun if a 20 ft tall tree casts a shadow of 16 yards?</p>
<p>A All Aboard</p> <p>A ship sailed from a port with a bearing of South 20° East. How far south (in miles) has the ship traveled after covering a distance of 100 km?</p>	<p>E Emergency</p> <p>A ladder on a fire truck can be turned to a maximum angle of 20° and can be extended to a maximum length of 100 feet. If the base of the ladder is mounted on the fire truck 2 yards above the ground, how high above the ground will the ladder reach?</p>
<p>H Happy Landing</p> <p>A plane is flying at an altitude of 20 km. The pilot wants to descend into an airport so that the path of the plane makes an angle of 16° with the ground. How far from the airport (horizontal distance) should the descent begin? (convert your answer to miles)</p>	<p>F F-I-R-E</p> <p>A forest ranger is 100 feet above the floor of the forest. He sees smoke rising below him. The angle of depression to the fire is 20°. What is the horizontal distance to the fire?</p>
<p>D Duck, Duck, Goose</p> <p>You are standing on a bridge that is 16 yards above the lake. There is a duck family standing on the dock that is 20 yards away from the base of the bridge. What is the angle of depression for your line of sight to the ducks?</p>	<p>B Bend in Tree</p> <p>An old 100 foot tall oak tree is snapped by a strong wind. If the break was 2 yards above the ground, what angle does the tree make at its bend?</p>

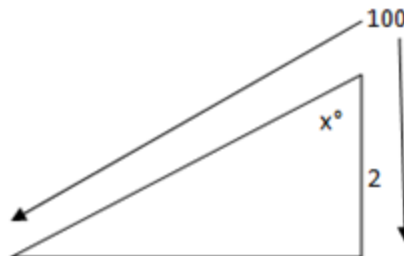
Diagram Cards for Matching Activity

Error Analysis Activity

B Bend in Tree

An old 100 foot tall oak tree is snapped by a strong wind. If the break was 2 yards above the ground, what angle does the tree make at its bend?



Review the solution below. Make any corrections that are needed. Explain the error that was made in the original work.

The handwritten solution includes a diagram of a right triangle with a hypotenuse labeled '100 ft' and an angle at the top labeled x° . The vertical side is labeled '2 yd' and '6 ft' with a diagonal line through it. A right angle symbol is at the bottom vertex. To the right of the diagram, the word 'Mark' is written.

Below the diagram, the student has written the following calculations:

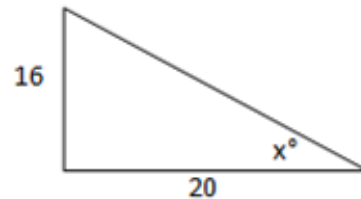
$$\cos^{-1}\left(\frac{6}{100}\right)$$

$$x^\circ = 86.6^\circ$$

Error Analysis Activity

D Duck, Duck, Goose

You are standing on a bridge that is 16 yards above the lake. There is a duck family standing on the dock that is 20 yards away from the base of the bridge. What is the angle of depression for your line of sight to the ducks?



Review the solution below. Make any corrections that are needed. Explain the error that was made in the original work.

Handwritten diagram of a right triangle. The vertical leg is labeled 16yd, the horizontal leg is labeled 20yd, and the angle at the bottom right is labeled x° . An angle of depression is indicated at the top vertex.

Kelly

$$\tan^{-1}\left(\frac{16}{20}\right)$$
$$x^\circ = 38.7^\circ$$

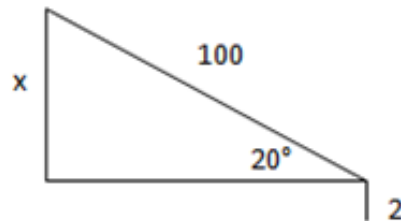
angle of depression

$$90 - 38.7^\circ = 51.3^\circ$$

Error Analysis Activity

E Emergency

A ladder on a fire truck can be turned to a maximum angle of 20° and can be extended to a maximum length of 100 feet. If the base of the ladder is mounted on the fire truck 2 yds above the ground, how high above the ground will the ladder reach?



Review the solution below. Make any corrections that are needed. Explain the error that was made in the original work.

Evan

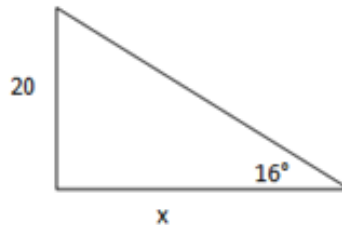
$$\cos 20 = \frac{x}{100}$$

$$x = 100 \cos 20 = 93.97 \text{ ft}$$

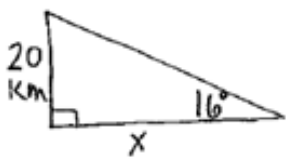
Error Analysis Activity

H Happy Landing

A plane is flying at an altitude of 20 km. The pilot wants to descend into an airport so that the path of the plane makes an angle of 16° with the ground. How far from the airport (horizontal distance) should the descent begin? (convert your answer to miles)



Review the solution below. Make any corrections that are needed. Explain the error that was made in the original work.



Mollie

$$\tan 16 = \frac{x}{20}$$

$$x = 20 \tan 16$$

$$x = 5.7 \text{ km}$$

$$5.7 \text{ km} \times 1.6 = 9.12 \text{ miles}$$

Situation 1

You are looking at a painting on the wall at the High Museum in Atlanta. You are standing 10 feet from the wall. Your angle of elevation to view the painting is 20° . (Your eyes are about 5 feet above the floor.)

Find the height of the painting.

Situation 2

You are looking 120 feet above the street on the rooftop of your hotel. You look down and see a McDonald's restaurant. The angle of depression is 70° .

Find the distance to McDonald's.

Student Directions

Part One: Matching Activity

- **Work with your partner to match each situation card with a diagram card.**
- **Discuss your matches with another group.**
- **Notify teacher that you are ready to explain your choices.**

Part Two: Analysis of Student Work

- **Work with your partner to analysis the mistakes made by each student.**